ON THE STABILITY OF GYROSCOPIC STABILIZERS

(OB USTOICHIVOSTI GIROSKOPICHESKIKH STABILIZATOROV)

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Power-assisted gyroscopic stabilizers [1-3] and platforms stabilized by means of floating (integrating) gyroscopes and servomotors [4,5] are sometimes used for the geometric stabilization of the orientation of one or several axes relative to inertial space [6,7]. In an investigation of the motion of such a system all its elements are normally assumed to be ideally balanced; dry friction is absent. The angles of rotation of the bodies of the system are employed as the determining coordinates. The generalized forces are the moments of the forces of viscous resistance and the moments of the servomotor about the axes of stabilization (axes of measurement). The latter are assumed functions of the angles of rotation of the gyroscope housings and sometimes also of their rates of change.

In the present paper the equations of motion of the gyroscopic-stabilization system as described are investigated - within the framework of the above generally-accepted conception of a mechanical model - for a case when the base of the stabilizer is fixed or has a translational motion relative to the inertial space. A criterion of the stability of the stabilizers of the above type in the Liapunov interpretation and in the presence of parametric disturbances is obtained. The effect of several categories of forces on the stability of a platform stabilized in space by means of two gyroscopes is examined. Sufficient conditions for the stability of a two-axes stabilizer are indicated.

1. Let $q_1, \ldots, q_n$ be the generalized coordinates of the system. Among them $q_{m+1}, \ldots, q_n$ are the angles of rotation of the corresponding bodies (of the platform, of the suspension frames) about the axes of stabilization; $q_1, \ldots, q_m$ $(1/2\, n \leqslant m \leqslant n)$ include the angles of rotation of the gyroscope housings $q_1, \ldots, q_l$ (from which the readings are taken);

the number of the latter is equal to the number of axes of stabilization $l = n - m$.

On the assumption that detachable devices of the potentiometer type are employed, we take the moments of the servomotor about the axes of stabilization to be holomorphic* functions of $q_1, \ldots, q_l, \dot{q}_1, \ldots, \dot{q}_n$

$$M_k = -\sum_{j=1}^{l} c_{kj}\, q_j - \sum_{j=1}^{n} b_{kj}''\, \dot{q}_j + M_k'(q_1, \ldots, q_l, \dot{q}_1, \ldots, \dot{q}_n) \qquad (k = m+1, \ldots, n)$$

where c_{kj}, b_{kj}'' are constants, M_k' are non-linearities.

The system is also subjected to dissipative forces with the dissipation function R; here R is a holomorphic function of $q_1, \ldots, q_n, \dot{q}_1, \ldots, \dot{q}_n$, the expansions of which begin with the constant-positive quadratic form

$$R^{(2)} = \frac{1}{2} \sum_{kj=1}^{n} b_{kj}\, \dot{q}_k\, \dot{q}_j \qquad (b_{kj} = b_{jk} = \text{const})$$

The equations of motion of the gyroscopic stabilizer can be written in the form

$$\frac{dq_k}{dt} = \dot{q}_k, \qquad \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_k}\right) - \frac{\partial T}{\partial q_k} = -\sum_{j=1}^{n} (g_{kj} + b_{kj})\, \dot{q}_j - \frac{\partial (R - R^{(2)})}{\partial \dot{q}_k} \qquad (k = 1, \ldots, m)$$

$$\frac{dq_k}{dt} = \dot{q}_k, \; \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_k}\right) - \frac{\partial T}{\partial q_k} = -\sum_{j=1}^{n} (g_{kj} + b_{kj} + b_{kj}'')\, \dot{q}_j - \tag{1.1}$$

$$-\sum_{j=1}^{l} c_{kj}\, q_j + M_k' - \frac{\partial (R - R^{(2)})}{\partial \dot{q}_k} \qquad (k = m+1, \ldots, n)$$

$$T = \frac{1}{2} \sum_{k,\, j=1}^{n} a_{kj}\, \dot{q}_k\, \dot{q}_j \qquad (a_{kj} = a_{jk})$$

Here T is the quadratic form of the velocity which is constant-positive when $q_1 = 0, \ldots, q_n = 0$; the quantities a_{kj}, $g_{kj} (g_{kj} = -g_{jk})$ are holomorphic functions of $q_1, \ldots, q_n$, where

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* Here, and subsequently, holomorphism and other properties of the functions are assumed within a certain small neighborhood of non-disturbed motion.

$$a_{kj}(0, \ldots, 0) \equiv a_{kj}^\circ, \quad g_{kj}(0, \ldots, 0) \equiv g_{kj}^\circ.$$

The problem on the stability of non-disturbed motion

$$q_1 = 0, \ldots, \quad q_n = 0, \quad \dot{q}_1 = 0, \ldots, \dot{q}_n = 0 \tag{1.2}$$

is posed.

The equations of the first approximation for the system (1.1)

$$\frac{dq_k}{dt} = \dot{q}_k, \quad \sum_{j=1}^{n} \left[a_{kj}^\circ \frac{d\dot{q}_j}{dt} + (g_{kj}^\circ + b_{kj})\, \dot{q}_j \right] = 0 \qquad (k = 1, \ldots, m)$$

$$\frac{dq_k}{dt} = \dot{q}_k, \sum_{j=1}^{n} \left[a_{kj}^\circ \frac{d\dot{q}_j}{dt} + (g_{kj}^\circ + b_{kj} + b_{kj}'')\, \dot{q}_j \right] + \sum_{j=1}^{l} c_{kj} q_j = 0$$

$$(k = m + 1, \ldots, n) \tag{1.3}$$

have a characteristic equation with an m-fold zero root. If among the remaining $n + l$ roots there is at least a single root with a positive real part, then the non-disturbed motion (1.2) is unstable. Otherwise, we are confronted by a special (according to Liapunov) case*.

Theorem 1. If the roots of the equation

$$\begin{vmatrix} \| a_{kj}^\circ \lambda^2 + (g_{kj}^\circ + b_{kj}) \lambda \| & \| a_{kj}^\circ \lambda + g_{kj}^\circ + b_{kj} \| \\ \cdots\cdots & \cdots\cdots \\ \| a_{kj}^\circ \lambda^2 + (g_{kj}^\circ + b_{kj} + b_{kj}'') \lambda + c_{kj} \| & \| a_{kj}^\circ \lambda + g_{kj}^\circ + b_{kj} + b_{kj}'' \| \end{vmatrix} = 0 \tag{1.4}$$

have negative real parts

$$\operatorname{Re} \lambda_k < 0 \qquad (k = 1, \ldots, n + l) \tag{1.5}$$

then the non-disturbed motion (1.2) of the system (1.1) is stable (relative to $q_1, \ldots, q_n, \dot{q}_1, \ldots, \dot{q}_n$). Any disturbed motion approximates

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* Therefore [8], the investigation of the stability of motion (1.2) must not be restricted to the study of this problem for linear equations (1.3) (as in [7]) nor the direct force effect of the floating gyroscopes on the platform be neglected (as in [6,5]). It is well known [3] that the equations of the theory of precession are also unsuitable for this purpose.

asymptotically to one of the motions

$$\dot{q}_1 = 0, \ldots, \dot{q}_n = 0, \quad q_1 = 0, \ldots, q_l = 0, \quad q_{l+1} = c_{l+1}, \ldots, q_n = c_n \quad (1.6)$$

The system (1.1) assumes m holomorphic Liapunov integrals not dependent on t:

$$\sum_{j=1}^{n} [(g_{kj}^\circ + b_{kj}) q_j + a_{kj}^\circ \dot{q}_j] + f_k(q_1, \ldots, q_n, \dot{q}_1, \ldots, \dot{q}_n) = A_k \quad (k = 1, \ldots, m) \quad (1.7)$$

where f_k contain no terms smaller than the second order and vanish if conditions (1.6) are fulfilled; $c_{l+1}, \ldots, c_n, A_1, \ldots, A_m$ are arbitrary constants, as small as desired if the initial disturbances are small.

In actual conditions, in addition to the forces taken into account in Equations (1.1), certain disturbing forces will act. The largest disturbing forces are those which give rise to moments about the axes of stabilization. The latter include, for instance, the moments of the force of gravity caused by inaccuracies in the balancing of the platform and the suspension gimbal rings. They are functions of the coordinates and parameters of the system (caused by the disturbances of the parameters [9]).

Let the coefficients in the expansions of the quantities which enter (1.1) and satisfy the requirements indicated earlier be holomorphic functions of the constant parameters $a_1, \ldots, a_i$ which describe the masses, moments of inertia, dissipation, etc.

Next, in accordance with the assumption, the disturbing forces $a_{i+k-m}\Phi_k (k = m+1, \ldots, n)$, where Φ_k are the holomorphic functions of $a_1, \ldots, a_i, q_1, \ldots, q_n$, are added to the right-hand side of the last group of equations in (1.1). They are brought about by the disturbances of the parameters $a_{i+1}, \ldots, a_{i+l}$, which may, for instance, denote a displacement of the centers of gravity of the platform and suspension rings from their axes of rotation.

Thus, a certain new model of the gyroscopic stabilizer has been conceived in the disturbed motion, which reflects more completely the properties of the physical choice under examination.

If $|c_{kj}|_1^l \neq 0$, then the system of equations

$$\sum_{j=1}^{l} c_{kj} q_j = M_k' (a_1, \ldots, a_i, q_1, \ldots, q_l, 0, \ldots, 0) + a_{i+k-m} \Phi_k \quad (k = m+1, \ldots, n)$$

determines l implicit functions

$$q_k = v_k(a_1, \ldots, a_{i+l}, \; q_{l+1}, \ldots, q_n) \qquad (k = 1, \ldots, l)$$

Theorem 2. If condition (1.5) is met, then the non-disturbed motion

$$\begin{aligned} q_1 &= 0, \ldots, q_n = 0, \qquad \dot q_1 = 0, \ldots, \; \dot q_n = 0, \\ a_1 &= \alpha_1, \ldots, a_i = \alpha_i, \quad a_{i+1} = 0, \ldots, a_{i+l} = 0 \end{aligned}$$

is stable (in relation to $q_1, \ldots, q_n, \dot q_1, \ldots, \dot q_n$) in the presence of parametric disturbances [9]. Any disturbed motion approaches asymptotically one of the equations

$$\dot q_1 = 0, \ldots, \dot q_n = 0, \quad q_{l+1} = c_{l+1}, \ldots, q_n = c_n, \quad q_k = v_k(a_1, \ldots, a_{i+l}, \; c_{l+1}, \ldots, c_n) \qquad (k = 1, \ldots, l) \tag{1.8}$$

The system of equations of motion admits m integrals of the type (1.7), but now f_k in them are holomorphic functions of $q_1, \ldots, q_n, \dot q_1, \ldots, \dot q_n, a_1 - \alpha_1, \ldots, a_i - \alpha_i, a_{i+1}, \ldots, a_{i+l}$ which contain no terms smaller than the second order and vanish if condition (1.8) is fulfilled. These integrals furnish evaluations for the disturbances $|q_k^\circ|$, $|\dot q_k^\circ|$, $|a_s - \alpha_s|$. If the conditions are satisfied, $|c_k|$ and $|v_k(a_1, \ldots, a_{i+l}, c_{l+1}, \ldots, c_n)|$ do not exceed the given positive quantities.

When $l = m = 1/2\, n$ and a stabilizer is being investigated having "gyroscopes with constant angular momenta", then normally

$$\begin{aligned} g_{kj}^\circ &= 0, \qquad a_{kj}^\circ = 0 \qquad (k \neq j, k, j = 1, \ldots, m), \\ b_{kj} &= 0, \qquad (k \neq j, \; k = 1, \ldots, n, \; j = 1, \ldots, m) \end{aligned}$$

To meet condition (1.5) it is necessary that

$$\begin{vmatrix} g_{1,m+1}^\circ & \cdots & g_{1n}^\circ \\ \cdots & \cdots & \cdots \\ g_{m,m+1}^\circ & \cdots & g_{mn}^\circ \end{vmatrix} \neq 0, \qquad |c_{kj}|_1^l \neq 0 \tag{1.9}$$

If the coefficients c_{kj}, b_{kj}'' are selected from the conditions

$$b_{kj}'' = a_{kj}^\circ \varkappa_j, \quad c_{kj} = g_{kj}^\circ \varkappa_j \qquad \left(\varkappa_j = \frac{b_{jj}}{a_{jj}^\circ} > 0\right) \quad \left(\begin{matrix} k = m+1, \ldots, n \\ j = 1, \ldots, l \end{matrix}\right) \tag{1.10}$$

then $\lambda_{n+1} = -\varkappa_1, \ldots, \lambda_{n+l} = -\varkappa_l$, while $\lambda_1, \ldots, \lambda_n$ is determined from the equation

$$\begin{vmatrix} \| a_{jj}^\circ \lambda \|_1^m & \| a_{kj}^\circ \lambda + g_{kj}^\circ \| \\ \dots & \dots \\ \| a_{kj}^\circ \lambda + g_{kj}^\circ \| & \| a_{kj}^\circ \lambda + g_{kj}^\circ + b_{kj} + b_{kj}'' \| \end{vmatrix} = 0 \qquad (1.11)$$

If $b_{kj} + b_{kj}'' = 0$ ($j, k = m + 1, \ldots, n$), then all of them will be purely imaginary [2,10]. But if

$$| b_{kj} + b_{kj}'' |_{m+1}^v > 0 \qquad (v = m + 1, \ldots, n)$$

which is normally the case, then it can be shown that

$$\operatorname{Re} \lambda_k < 0 \qquad (k = 1, \ldots, n)$$

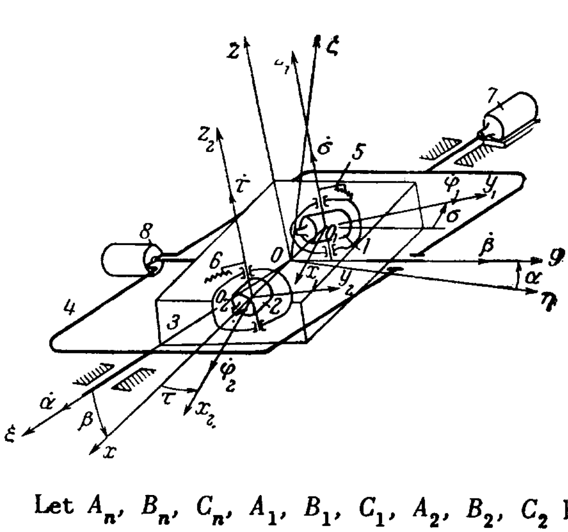
For stabilizers having gyroscopes with variable angular momenta when the mechanical properties of the gyromotors, which are rigidly coupled, can be formulated as follows:

$$M_k = - b_k \dot{q}_k + M'_k \qquad (b_k > 0, \quad k = l + 1, \ldots, m) \qquad (1.12)$$

where $\dot{q}_{l+1}, \ldots, \dot{q}_m$ is the disturbance of the velocities of rotation of the gyroscopes about their own axes and M_k' are the non-linearities, the problem is solved in a similar manner. In the case of condition (1.10) and complete dissipation

$$\operatorname{Re} \lambda_k < 0 \qquad (k = 1, \ldots, n + l)$$

2. We will investigate the platform [7] with two axes, stabilized in space by means of two gyroscopes and motors controlling them, shown in the figure, where $O\xi\eta\zeta$ is the inertial system of coordinates; $Oxyz$, $O_1x_1y_1z_1$, $O_2x_2y_2z_2$ are the coordinates of the principal axes of inertia of the platform 3 (and of the rotor 8 of the motor rigidly connected with it) and of the housings 1 and 2 of the gyromotors; in the figure, 5 and 6 are potentiometer-type detachable devices, 7 and 8 are servomotors.



Let A_n, B_n, C_n, A_1, B_1, C_1, A_2, B_2, C_2 be the moments of inertia of the platform and the gyromotors relative to these axes. The axes O_1y_1 and O_2x_2 of rotation of the gyroscopes about their own axes move in the plane of the platform Oxy. We denote the distance OO_1 by l_1. We also introduce the following notation:

$$C \equiv C_n + C_1 + C_2 + 2ml_1^2$$

$$A_x(\sigma, \tau) \equiv A_n + A_1 \cos^2\sigma + B_1 \sin^2\sigma + A_2 \cos^2\tau + B_2 \sin^2\tau$$

$$A_\xi(\beta, \sigma, \tau) \equiv A_x(\sigma, \tau)\cos^2\beta + C\sin^2\beta + A''$$

$$B(\sigma, \tau) \equiv B_n + 2ml_1^2 + A_1 \sin^2\sigma + B_1 \cos^2\sigma + A_2 \sin^2\tau + B_2 \cos^2\tau$$

$$I_{\alpha\beta}(\sigma, \tau) \equiv (A_1 - B_1)\sin\sigma\cos\sigma + (A_2 - B_2)\sin\tau\cos\tau$$

where A'' is the moment of inertia of the frame 4 (and of the bodies connected with it) relative to the axis $O\xi$; m is the mass of a gyromotor; ϕ_1, ϕ_2 are the angles of rotation of the gyroscopes about their own axes; $H_1 = I_1\dot{\phi}_1^\circ$, $H_2 = I_2\dot{\phi}_2^\circ$ are their non-disturbed angular momenta

$$\xi_1 = \varphi_1 - \dot{\varphi}_1^\circ t, \quad \xi_2 = \varphi_2 - \dot{\varphi}_2^\circ t, \quad \alpha' = \alpha - \alpha_0, \quad \beta' = \beta - \beta_0$$

We write down the equations of motion in the Lagrange form with the mass of all the bodies of the system taken into account:

$$\begin{gathered} C_1\ddot{\sigma} + C_1\ddot{\alpha}\sin\beta + (A_1 - B_1)(\dot{\alpha}^2\cos^2\beta - \dot{\beta}^2)\sin\sigma\cos\sigma + \\ + [C_1 - (A_1 - B_1)\cos 2\sigma]\,\dot{\alpha}\,\dot{\beta}\cos\beta + I_1\,\dot{\xi}_1\,(\dot{\beta}\sin\sigma + \dot{\alpha}\cos\beta\cos\sigma) \\ = -H_1(\dot{\beta}\sin\sigma + \dot{\alpha}\cos\beta\cos\sigma) + M_{z1} \end{gathered}$$

$$\begin{gathered} C_2\ddot{\tau} + C_2\ddot{\alpha}\sin\beta + [C_2 - (A_2 - B_2)\cos 2\tau]\,\dot{\alpha}\dot{\beta}\cos\beta + (A_2 - B_2) \times \\ \times (\dot{\alpha}^2\cos^2\beta - \dot{\beta}^2)\sin\tau\cos\tau + I_2\,\dot{\xi}_2\,(\dot{\alpha}\cos\beta\sin\tau - \dot{\beta}\cos\tau) \\ = -H_2(\alpha\cos\beta\sin\tau - \dot{\beta}\cos\tau) + M_{z2} \end{gathered}$$

$$I_1(\ddot{\xi}_1 + \ddot{\beta}\cos\sigma - \ddot{\alpha}\cos\beta\sin\sigma - \dot{\beta}\dot{\sigma}\sin\sigma + \dot{\alpha}\dot{\beta}\sin\beta\sin\sigma - \dot{\alpha}\dot{\sigma}\cos\beta\cos\sigma) = M_1$$

$$I_2(\ddot{\xi}_2 + \ddot{\beta}\sin\tau + \ddot{\alpha}\cos\beta\cos\tau + \dot{\beta}\dot{\tau}\cos\tau - \dot{\alpha}\dot{\beta}\sin\beta\cos\tau - \dot{\alpha}\dot{\tau}\cos\beta\sin\tau) = M_2$$

$$\begin{gathered} A_\xi(\beta, \sigma, \tau)\,\ddot{\alpha} + I_{\alpha\beta}(\sigma, \tau)\cos\beta\ddot{\beta} + C_1\sin\beta\ddot{\sigma} + C_2\sin\beta\ddot{\tau} - I_1\ddot{\xi}_1\cos\beta\sin\sigma + \\ + I_2\ddot{\xi}_2\cos\beta\cos\tau + \dot{A}_\xi(\beta, \sigma, \tau)\,\dot{\alpha} + \dot{I}_{\alpha\beta}(\sigma, \tau)\cos\beta\dot{\beta} - I_{\alpha\beta}(\sigma, \tau)\sin\beta\dot{\beta}^2 + \\ + C_1\cos\beta\dot{\beta}\dot{\sigma} + C_2\cos\beta\dot{\tau}\dot{\beta} - I_1\dot{\xi}_1\dot{\sigma}\cos\beta\cos\sigma + \\ + I_1\dot{\xi}_1\dot{\beta}\sin\beta\sin\sigma - I_2\dot{\xi}_2\dot{\beta}\sin\beta\cos\tau - I_2\dot{\xi}_2\dot{\tau}\cos\beta\sin\tau \\ = H_1(\dot{\sigma}\cos\beta\cos\sigma - \dot{\beta}\sin\beta\sin\sigma) + H_2(\dot{\beta}\sin\beta\cos\tau + \dot{\tau}\cos\beta\sin\tau) + M_\xi \end{gathered}$$

$$B(\sigma,\tau)\ddot{\beta}+I_{\alpha\beta}(\sigma,\tau)\cos\beta\ddot{\alpha}+I_1\ddot{\xi}_1\cos\sigma+I_2\ddot{\xi}_2\sin\tau+\dot{B}(\sigma,\tau)\dot{\beta}+$$
$$+\dot{I}_{\alpha\beta}(\sigma,\tau)\cos\beta\dot{\alpha}+[A_x(\sigma,\tau)-C]\dot{\alpha}^2\sin\beta\cos\beta-C_1\dot{\alpha}\dot{\sigma}\cos\beta-C_2\dot{\alpha}\dot{\tau}\cos\beta-$$
$$-I_1\dot{\xi}_1(\dot{\sigma}+\dot{\alpha}\sin\beta)\sin\sigma+I_2\dot{\xi}_2(\dot{\tau}+\dot{\alpha}\sin\beta)\cos\tau=H_1(\dot{\sigma}+\dot{\alpha}\sin\beta)\sin\sigma-$$
$$-H_2(\dot{\tau}+\dot{\alpha}\sin\beta)\cos\tau+M_y$$

We will investigate the stability of the following positions of equilibrium of the platform (of the permanent rotations of the gyroscopes):

$$\sigma=0,\quad \tau=0,\quad \alpha=\alpha_0,\quad \beta=\beta_0,\quad \dot{\sigma}=0,\quad \dot{\tau}=0,\quad \dot{\alpha}=0,$$
$$\dot{\beta}=0,\quad \dot{\xi}_1=0,\quad \dot{\xi}_2=0\ \left(|\beta_0|<\tfrac{\pi}{2}\right) \qquad (2.1)$$

When the moments about the axes of rotation are zero (all bodies of the system ideally balanced, the servomotors 7 and 8 disconnected), then, in first approximation, the equilibria (2.1) are stable. The frequencies of free vibrations are determined by the roots of the characteristic equation

$$\lambda_{1,2}{}^{\circ}=\frac{\pm iH_1\cos\beta_0}{\sqrt{A^*(\beta_0)C_1}},\qquad \lambda_{3,4}{}^{\circ}=\frac{\pm iH_2}{\sqrt{B^*C_2}}\qquad (i=\sqrt{-1})$$

where

$$A^*(\beta_0)\equiv A_\xi(\beta_0,0,0)-(C_1+C_2)\sin^2\beta_0-I_2\cos^2\beta_0,\qquad B^*\equiv B(0,0)-I_1$$

In practice, induction motors are used as gyromotors, the mechanical properties of which are determined by Expressions (1.12); there is then complete dissipation under these conditions. In this case, any position of equilibrium (2.1) is stable. The free oscillations of the platform decay [11]. Nevertheless, it can be shown in a similar manner [11] that any position of equilibrium (2.1) is unstable when there are parametric disturbances caused by inaccuracies in the balancing of the platform and frame. The need arises for the introduction into the system of stabilizing motors.

When the servomotors are switched on, let the moments be expressed as

$$M_{z_1}=-b_\sigma\dot{\sigma}+M_{z_1}{}',\qquad M_{z_2}=-b_\tau\dot{\tau}+M_{z_2}{}'$$
$$M_y=-b_\beta\dot{\beta}-k_2\tau+M_y{}',\ M_\xi=-b_\alpha\dot{\alpha}+k_1\sigma-b_{\alpha\sigma}{}''\dot{\sigma}-b_{\alpha\tau}{}''\dot{\tau}+M_\xi{}'$$

where the constants k_1, k_2, b_σ, b_τ are positive and b_β, b_α, $b_{\alpha\sigma}''$, $b_{\alpha\tau}''$ are not negative. The mechanical properties of the gyromotors are assumed such that absolutely rigid coupling exists ($\dot{\phi}_1\equiv\dot{\phi}_1{}^{\circ}$, $\dot{\phi}_2\equiv\dot{\phi}_2{}^{\circ}$, [7]).

If with $\beta_0=0$ ($b_{\alpha\tau}''=b_{\alpha\sigma}''=0$) the following conditions are met:

$$\left(\frac{b_\sigma}{C_1} + \frac{b_\alpha}{A_\xi(0,0,0)}\right)(b_\alpha b_\sigma + H_1^2) > H_1 k_1 > 0$$

$$\left(\frac{b_\tau}{C_2} + \frac{b_\beta}{B(0,0)}\right)(b_\tau b_\beta + H_2^2) > H_2 k_2 > 0 \qquad (2.2)$$

or, with a certain arbitrary $\beta_0(b_\alpha > 0,\ b_\beta > 0)$

$$k_1 = \frac{H_1 b_\sigma}{C_1}\cos\beta_0, \quad b_{\alpha\sigma}'' = b_\sigma \sin\beta_0, \quad k_2 = \frac{H_2 b_\tau}{C_2}, \quad b_{\alpha\tau}'' = b_\tau \sin\beta_0 \qquad (2.3)$$

then the corresponding equilibrium (2.1) is stable (asymptotically with respect to σ, τ, $\dot\sigma$, $\dot\tau$, $\dot\alpha$, $\dot\beta$). The stability is preserved in the case of parametric disturbances of the balancing of the platform and frame. Any disturbed motion approaches one of the equilibria

$$\sigma = \sigma_\infty, \qquad \tau = \tau_\infty, \qquad \alpha' = \alpha_\infty', \qquad \beta' = \beta_\infty'.$$

Note. If in conditions (2.3) $b_\alpha = 0$, $b_\beta = 0$, then the roots of the equations of the type (1.4) are equal to

$$\lambda_{1,2} = \frac{\pm iH_1\cos\beta_0}{\sqrt{(A^*(\beta_0) + I_2\cos^2\beta_0)\,C_1}}, \quad \lambda_{3,4} = \frac{\pm iH_2}{\sqrt{B(0,0)\,C_2}}, \quad \lambda_5 = -\frac{b_\sigma}{C_1}, \quad \lambda_6 = -\frac{b_\tau}{C_2}$$

We assume the plane of the platform horizontal when $\alpha = 0$, $\beta = 0$, and formulate the force function due to gravity after the disturbance of the coordinates of the centers of gravity of the platform and frame as follows:

$$U = -r_1\sin(\alpha + \gamma) - r_2\cos\alpha\cos(\beta + \delta)$$

where r_1, r_2, γ, δ are constants; and then we obtain the estimates

$$|\sigma_\infty| < \frac{\eta}{k_1}(1 + |\sin\alpha_0| + O(\eta)) \leqslant \frac{2\eta}{k_1}, \qquad |\tau_\infty| < \frac{\eta}{k_2}$$

$$|\alpha'_\infty| < |\alpha'^\circ| + \frac{\eta}{H_1\cos\beta_0}\left[b_\sigma\left(1 + \frac{1 + |\sin\alpha_0|}{k_1}\right) + C_1(1 + |\sin\beta_0|) + O(\eta)\right]$$

$$|\beta_\infty'| < |\beta'^\circ| + \frac{\eta}{H_2}\left[b_\tau\left(1 + \frac{1}{k_2}\right) + C_2(1 + |\sin\beta_0|) + O(\eta)\right]$$

where η is the largest modulus of the disturbances (in non-dimensional form)

$$\sigma^\circ, \tau^\circ, \alpha'^\circ, \beta'^\circ, \dot\sigma^\circ, \dot\tau^\circ, \dot\alpha^\circ, \dot\beta^\circ, a_1 - \alpha_1, \ldots, a_i - \alpha_i, r_1, r_2$$

A comparison of these estimates shows that the selection of parameters (2.3) may prove to be close to the optimum.

A calculation of the variability of the angular momenta of the gyroscopes, caused by the non-absolute rigidity in the mechanical properties of the gyromotors, as in (1.12), shows that in the case of conditions (2.2) or (2.3) the stability is preserved.

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